Confirmatory factor analysis of a rubric for assessing algorithmic thinking on undergraduate students

Análisis factorial confirmatorio de una rúbrica para evaluar pensamiento algorítmico en estudiantes universitarios

Análise fatorial confirmatória de uma rubrica para avaliar o pensamento algorítmico em estudantes universitários

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Abstract

Algorithmic thinking is a key element for individuals to be aligned with the computer era. Its study is important not only in the context of computer science but also in mathematics education and all STEAM contexts. However, despite its importance, a lack of research treating it as an independent construct and validating its operational definitions or rubrics to assess its development in university students through confirmatory factor analysis has been discovered. The aim of this paper is to conduct a construct validation through confirmatory factor analysis of a rubric for the algorithmic thinking construct, specifically to measure its level of development in university students. Confirmatory factor analysis is performed on a series of models based on an operational definition and a rubric previously presented in the literature. The psychometric properties of these models are evaluated, with most of them being discarded. Further research is still needed to expand and consolidate a useful operational definition and the corresponding rubric to assess algorithmic thinking in university students. However, the confirmatory factor analysis confirms the construct validity of the rubric, as it exhibits very good psychometric properties and leads to an operational definition of algorithmic thinking composed of four components: Problem analysis, algorithm construction, input case identification, and algorithm representation.

Keywords: algorithmic thinking, higher education students, educational assessment, confirmatory factor analysis, STEM education.

Resumen

El pensamiento algorítmico es un elemento clave para ser un individuo alineado con la era de la computación. Su estudio es importante no solo en el contexto de las ciencias de la computación, sino también en la didáctica de la matemática y en todos los contextos STEAM. Pero a pesar de su importancia, se ha descubierto una carencia de investigaciones que lo traten como un constructo independiente y que validen sus definiciones operacionales o rúbricas para evaluar su desarrollo en estudiantes universitarios mediante análisis factorial confirmatorio. El objetivo de este artículo es realizar una validación de constructo por medio de análisis factorial confirmatorio de una rúbrica para el constructo pensamiento algorítmico, específicamente para medir su nivel de desarrollo en estudiantes universitarios. Se realiza un análisis factorial confirmatorio sobre una serie de modelos basados en una definición operacional y una rúbrica previamente presentadas en la literatura. Se evalúan las propiedades psicométricas de estos modelos, descartándose la mayoría de ellos. Aún se necesita más investigación para ampliar y consolidar una definición operacional útil, y la rúbrica correspondiente, para evaluar el pensamiento algorítmico en estudiantes universitarios. Sin embargo, el análisis factorial confirmatorio llevado a cabo confirma la validez de constructo de la rúbrica, ya que presenta muy buenas propiedades psicométricas y conduce a una definición operacional de pensamiento algorítmico compuesta por cuatro componentes: análisis del problema, construcción del algoritmo, identificación de los casos de entrada y representación del algoritmo.

Resumo

O pensamento algorítmico é um elemento-chave para que os indivíduos estejam alinhados com a era da computação. Seu estudo é importante não apenas no âmbito da ciência da computação, mas também na didática da matemática e em todos os contextos STEAM. No entanto, apesar de sua importância, faltam pesquisas que o tratem como um construto independente e que validem suas definições operacionais ou rubricas para avaliar seu desenvolvimento em estudantes universitários por meio de análises fatoriais confirmatórias. O objetivo deste artigo é realizar uma validação de construto por meio de análise fatorial confirmatória de uma rubrica para o construto pensamento algorítmico, especificamente para medir seu nível de desenvolvimento em estudantes universitários. É realizada uma análise fatorial confirmatória sobre uma série de modelos baseados em uma definição operacional e uma rubrica previamente apresentadas na literatura. As propriedades psicométricas desses modelos são avaliadas, e a maioria deles é descartada. Ainda são necessárias mais pesquisas para ampliar e consolidar uma definição operacional útil e a rubrica correspondente para avaliar o pensamento algorítmico em estudantes universitários. No entanto, a análise fatorial confirmatória realizada confirma a validade de construto da rubrica, pois esta apresenta propriedades psicométricas muito boas, e conduz a uma definição operacional de pensamento algorítmico composta por quatro componentes: análise do problema, construção do algoritmo, identificação dos casos de entrada, e representação do algoritmo.

Introduction

Computational thinking (CT) is becoming a fundamental skill for 21st-century citizens worldwide (Grover & Pea, 2018; Nordby et al., 2022) since it is related to beneficial skills that are considered applicable in everyday life (Wing, 2006; 2017). Algorithmic thinking (AT) is considered the main component of CT (Juškevičienė, 2020; Selby & Woollard, 2013; Stephens & Kadijevich, 2020). Moreover, most of the CT definitions have their roots in AT (Juškevičienė, 2020), although some very influential CT definitions do not mention AT at all (Shute et al., 2017; Weintrop et al., 2016; Wing, 2006).

Today, daily life is surrounded by algorithms and governed by algorithms, so AT is considered one of the key elements to be an individual aligned with the era of computing (Juškevičienė, 2020). Research on AT is very important in computer science education, but it also has a vital role in mathematics education and STEAM contexts (Kadijevich et al., 2023). Despite its importance, AT has been found to lack research that treats it as an independent construct with independent-of-CT assessments (Park & Jun, 2023).

CT is still a blurry psychological construct, and its assessment remains a thorny, unresolved issue (Bubica & Boljat, 2021; Martins-Pacheco et al., 2020; Román-González et al., 2019; Tang et al., 2020), and open, as a research challenge, demanding scholars' attention urgently (Poulakis & Politis, 2021). Moreover, the same happens with AT (Lafuente Martínez et al., 2022; Stephens & Kadijevich, 2020) due to its relationship with CT.

A wide variety of CT assessment tools are available (Tang et al., 2020; Zúñiga Muñoz et al., 2023), ranging from diagnostic tools to measures of CT proficiency and assessments of perceptions and attitudes towards this thinking way, among others. (Román-González et al., 2019). However, empirical research evaluating the validity and reliability of these instruments remains relatively low compared to the volume of research in this area (Tang et al., 2020). Consequently, as noted by Bubica and Boljat (2021), "there is still not enough research on CT evaluation to provide teachers with enough support in the field" (p. 453).

While much research focuses on measuring and assessing CT (Poulakis & Politis, 2021), the same cannot be said for AT. Moreover, these research streams do not converge (Stephens & Kadijevich, 2020). Furthermore, a limited percentage of research on CT assessment is directed towards undergraduate students (Tang et al., 2020).

A limited number of efforts, albeit divergent, have been made to formulate operational definitions for AT, as evidenced by works such as those by Juškevičienė and Dagienė (2018), Lafuente Martínez et al. (2022), Navas-López (2021), and Park and Jun (2023). Additionally, attempts have been undertaken to develop an assessment rubric (Navas-López, 2021) and apply confirmatory factor analysis (CFA) for AT measurement instruments (Bubica & Boljat, 2021; Lafuente Martínez et al., 2022).

Despite these contributions, there remains a notable gap in the literature regarding validating instruments specifically designed to assess AT as an independent construct distinct from CT. So far, there are only two factorial models for AT in adolescents or adults with acceptable psychometric properties: a unifactorial model (Lafuente Martínez et al., 2022) and a bifactorial model (Ortega Ruipérez et al., 2021). Therefore, this research endeavors to develop a more detailed factorial model through the

construct validation of the rubrics applied by Navas-López (2021) to assess AT in undergraduate students.

Developing reliable instruments for studying AT is crucial, particularly with the rising integration of CT and AT in basic education curricula around the world (Kadijevich et al., 2023). Continuous research is crucial to identify components, dimensions, and factors that illuminate the assessment of AT (Lafuente Martínez et al., 2022; Park & Jun, 2023).

Literature review

Some definitions

As this paper focuses on AT, it is crucial to understand the concept of an algorithm clearly. According to Knuth (1974), an algorithm is defined as "a precisely-defined sequence of rules instructing how to generate specified output information from given input information within a finite number of steps" (p. 323). This definition encompasses human and machine execution without specifying any particular technology requirement.

In line with this definition, Lockwood et al. (2016) describe AT as "a logical, organized way of thinking used to break down a complicated goal into a series of (ordered) steps using available tools" (p. 1591). This definition of AT, like the previous definition of algorithm, does not require the intervention of any specific technology.

For Futschek (2006), AT is the following set of skills that are connected to the construction and understanding of algorithms:

    a.   the ability to analyze given problems,

    b.   the ability to specify a problem precisely,

    c.   the ability to find the basic actions that are adequate to the given problem,

    d.   the ability to construct a correct algorithm for a given problem using the basic actions,

    e.   the ability to think about all possible special and regular cases of a problem,

    f.   the ability to improve the efficiency of an algorithm. (p. 160)

There are other operational definitions for AT. Some of them emphasize specific skills, such as the correct implementation of branching and iteration structures (Grover, 2017; Bubica & Boljat, 2021). Most of them describe AT in terms of a list of skills (Bubica & Boljat, 2021; Lafuente Martínez et al., 2022; Lehmann, 2023; Park & Jun, 2023; Stephens & Kadijevich, 2020), such as analyzing algorithms or creating sequences of steps. Despite similarities, these skill lists cannot be considered equivalent to each other.

The definition of CT will now be addressed by expanding on the concept of AT. It could be defined as "the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer –human or machine– can effectively carry out" (Wing, 2017, p. 8). Unlike the AT definitions, this CT definition allows for machines' involvement. Consequently, CT does involve considerations about the technology underlying the execution of these solutions (Navas-López, 2024).

As with AT, different operational definitions of CT describe it as a list of various components or skills, such as abstraction, decomposition, and generalization (Otero Avila et al., 2019; Tsai et al., 2022). Additionally, several authors explicitly include AT as an operational component of CT in their empirical research (Bubica & Boljat, 2021; Korkmaz et al., 2017; Lafuente Martínez et al., 2022; Otero Avila et al., 2019; Tsai et al., 2022).

For Stephens and Kadijevich (2020), the cornerstones of AT are decomposition, abstraction, and algorithmization, whereas CT incorporates these elements along with automation. This distinction underscores that automation is the defining factor separating AT from CT.

Lehmann (2023) describes algorithmization as the ability to design a set of ordered steps to produce a solution or achieve a goal. These steps include inputs and outputs, basic actions, or algorithmic concepts such as iterations, loops, and variables. Note the similarity with Lockwood et al. (2016) definition of AT.

Kadijevich et al. (2023) assert that AT requires distinct cognitive skills, including abstraction and decomposition. Following Juškevičienė and Dagienė (2018), decomposition involves breaking down a problem into parts (sub-problems) that are easier to manage, while abstraction entails identifying essential elements of a problem or process, which involves suppressing details and making general statements summarizing particular examples. Furthermore, these two skills are present in many CT operational definitions (Bubica & Boljat, 2021; Juškevičienė & Dagienė, 2018; Lafuente Martínez et al., 2022; Martins-Pacheco et al., 2020; Otero Avila et al., 2019; Selby & Woollard, 2013; Shute et al., 2017).

Unsurprisingly, the most commonly assessed CT components are algorithms, abstraction, and decomposition (Martins-Pacheco et al., 2020), which closely aligns with the AT conception by Stephens and Kadijevich (2020). In fact, in the study conducted by Lafuente Martínez et al. (2022), the researchers aimed to validate a test for assessing CT in adults, avoiding technology (or automation). However, their CFA results suggest a simpler CT concept, governed by a single ability associated with recognizing and expressing routines to address problems or tasks, akin to systematic, step-by-step instructions—essentially, AT.

The distinction between AT and CT remains to be clarified in current scientific literature. Nonetheless, this study adopts Stephens and Kadijevich's (2020) perspective, emphasizing that the primary difference lies in automation. Specifically, AT excludes broader aspects of technology use and social implications (Navas-López, 2024). The study also focuses on abstraction and decomposition as integral components of AT.

Algorithmic thinking and Computational thinking assessments

According to Bubica and Boljat (2021), "to evaluate CT, it is necessary to find evidence of a deeper understanding of a CT-relevant problem solved by a pupil, that is, to find evidence of understanding how the pupil created their coded solution" (p. 428). Grover (2017) recommends the use of open-ended questions for making "systems of assessments" to assess AT.

Bacelo and Gómez-Chacón (2023) emphasize the significance of unplugged activities for observing students' skills and behaviors, and identifying patterns that can reveal

strengths and weaknesses in AT learning, as shown by Lehmann (2023). Empirical data suggests that students engaged in unplugged activities demonstrate marked improvements in CT skills compared to those in plugged activities (Kirçali & Özdener, 2023). Unplugged activities are generally effective in fostering CT skills (Chen et al., 2023), making them essential components in AT assessment instruments to identify patterns that reveal strengths and weaknesses.

There is a common tendency to assess AT and CT through small problem-solving tasks, often using binary criteria like 'solved/unsolved' or 'correct/incorrect' without detailed rubrics for each problem. Examples can be found across different age groups: young children (Kanaki & Kalogiannakis, 2022), K-12 education (Oomori et al., 2019; Ortega et al., 2021), and adults (Lafuente Martínez et al., 2022). Conversely, there are proposals to qualitatively assess CT using unplugged complex problems, focusing on students' cognitive processes (Lehmann, 2023).

In addition, Bubica and Boljat (2021) suggest adapting problem difficulty to students' level, because assessments for AT and CT vary in effectiveness for different learners (Grover, 2017). Therefore, an effective AT assessment should include unplugged, preferably open-ended problems adjusted to students' expected levels to identify cognitive processes better.

Validity of CT/AT instruments through factor analysis

According to Lafuente Martínez et al. (2022), assessments designed to measure CT in adults, as discussed in the literature, often lack substantiated evidence concerning crucial validity aspects, particularly the internal structure and test content. Some studies validate CT instruments, which include the AT construct, through factor analysis, but they have problems with psychometric properties, such as those of Ortega Ruipérez and Asensio Brouard (2021), Bubica and Boljat (2021) and Sung (2022).

Ortega Ruipérez and Asensio Brouard (2021) shift the focus of CT assessment towards problem-solving to measure the performance of cognitive processes, moving away from computer programming and software design. Their research aims to validate an instrument for assessing CT through problem-solving in students aged 14-16 using CFA. However, the instrument exhibits low factor loadings, including one negative. By emphasizing problem-solving outside the realm of computer programming, this instrument aligns more closely with the interpretation of AT by Stephens and Kadijevich (2020), as it removes automation from CT. Nevertheless, its results are pretty poor, as the conducted CFA only identifies two independent factors: problem representation and problem-solving.

Bubica and Boljat (2021) applied an exploratory factor analysis for a CT instrument, special for the Croatian basic education curriculum, with a mix of simple answer questions and open-ended problems in students aged 11-12. AT evaluation criteria focus on sequencing, conditionals, and cycles. However, the factor loadings are low, and the grouping of the tasks (items) according to the factor analysis is strangely overloaded towards one factor.

Sung (2022) conducted a validation through CFA for two measurements to assess CT in young children aged 5 and 6 years. It has certain limitations, including the weak factor loadings of specific items, suboptimal internal consistency in subfactors, and low internal reliability. Furthermore, Sung (2022) reflects that "young children who are

still cognitively developing and lack specific high-level thinking skills seem to be at a stage before CT's major higher-order thinking functions are subdivided" (p. 12992). This may be the same problem in Bubica and Boljat's (2021) study results.

Lafuente Martínez et al. (2022) performed a CFA for a CT instrument for adults (average 23.58 years), and they did not find multidimensionality in their evidence, thus failing to confirm the assumption of difference between CT and AT.

Other CFA studies focus on CT instruments with strong psychometric properties, but they primarily assess disposition toward CT rather than problem-solving abilities. For instance, Tsai et al. (2022) used CFA to validate their 19-item questionnaire, demonstrating good item reliability, internal consistency, and construct validity in measuring CT disposition. Their developmental model highlighted that decomposition and abstraction are key predictors of AT, evaluation, and generalization, suggesting their critical role in CT development.

So, there is a lack of CFA-specialized studies on AT published in the last five years, independent of CT, focused solely on problem solving, and on university/college undergraduate students (young adults).

Rubric for algorithmic thinking

According to Bubica and Boljat (2021), algorithmic solutions are always difficult to evaluate because "in the process of creating a model of evidence, it is crucial to explore all possible evidence of a pupil's knowledge without losing sight of the different ways in which it could be expressed within the context and the requirements of the task itself" (p. 442). Therefore, one way to assess students' performance objectively and methodically is through a rubric (Chowdhury, 2019).

Furthermore, it is highly recommended to construct rubrics to assess students' learning, cognitive development, or skills, based on Bloom's taxonomy of educational objectives (Moreira Gois et al., 2023; Noor et al., 2023). This taxonomy of educational objectives includes Knowledge (knowing), understanding, application, analysis, synthesis, and evaluation (Bloom & Krathwohl, 1956).

Although some more or less precise recent operational definitions have been proposed for AT (Juškevičienė & Dagienė, 2018; Lafuente Martínez et al., 2022; Park & Jun, 2023), these do not include a specific (or general) rubric to assess performance levels for each of the components or factors these operational definitions claim to compose the AT construct.

The only rubric to measure AT development found, independent of CT, based on open-ended problem solving for undergraduate students, is that by Navas-López's (2021) master thesis. This study has a correlational scope and does not include construct validation for the rubric.

Navas-López (2021) proposed two operationalizations for AT: One operationalization is for a beginner's AT level, and the other is for an expert's AT level. In the skills list that composes a beginner's AT, the educational objectives of Knowledge (knowing), understanding and application (execution) of algorithms have been included. Table 1 shows the latent variables (components or factors) and observable variables of the operationalization for a beginner's AT, according to this proposal, and Figure 1 shows the corresponding structural model.
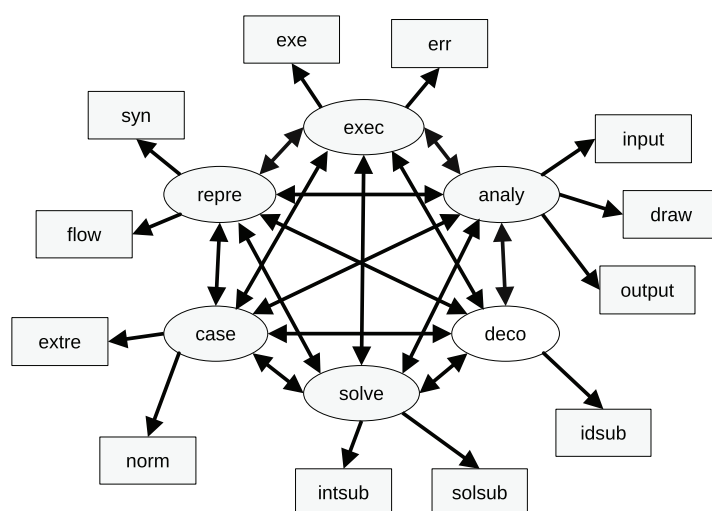
Table 1

Generic operationalization of a beginner's algorithmic thinking

Components (factors)	Sub-components (observable variables)	Code
Problem analysis	Identifies inputs	input
	Makes a useful graphic or diagram	draw
	Identifies outputs	output
Decomposition of the problem into sub-problems	Identifies the sub-problems that compose the problem	idsub
Construction of a correct algorithm that solves the problem	Implements a solution to the identified sub-problems	solsub
	Integrates the different partial solutions to solve the complete problem	intsub
Identification of normal and special cases	Identifies normal cases	norm
	Identifies special or extreme cases	extre
Construction of a formal representation for the algorithm that solves the problem	Represents the execution flow. In the case of using pseudocode, this includes indentation and nesting of control structures such as if-then-else, do-while, for-each, etc.). In the case of flowcharts, the flow must be represented in an unambiguous way.	flow
	Uses appropriate symbology and syntactic rules (such as diamonds for conditional branches in flowcharts or "reserved words" in pseudocode).	syn
Executing an algorithm	Effectively executes an algorithm given its representation	run
	Executes an algorithm that is considered potentially correct in order to check normal and special cases, and look for the causes of errors already identified.	err

Note. Translation of Table 3.3 from Navas-López (2021, p. 60)

Figure 1

Structural model of a beginner's algorithmic thinking operationalization

Note. Own elaboration from factors and observable variables from Table 1.

Navas-López (2021) explains that he primarily used the definition provided by Futschek (2006) due to its wide referencing in multiple publications on AT and CT. However, he supplemented his operational definition with the definition by Grozdev and Terzieva (2015), particularly concerning problem decomposition, the relationship between sub-problems, and the formalization of algorithm representation. Finally, the notion of algorithm debugging from Sadykova and Usolzev (2018) was incorporated.

This proposal also incorporates the components outlined by Stephens and Kadijevich (2020): Abstraction in problem analysis and identification of sub-problems, decomposition explicitly, and algorithmization, as understood by Lehmann (2023).

Navas-López (2021) also developed a generic analytic rubric to assess a beginner's AT level from its factors for a given problem, following the steps of Mertler's (2001) scoring rubric design and the template presented by Cebrián de la Serna and Monedero Moya (2014). Table 2 shows the detailed rubric.

Table 2
Generic analytic rubric to assess a beginner's algorithmic thinking level.

Variable	Indicators / Evidence				
input	Does not identify any input variable [0%]	Defines/uses other variable(s) as input variables [10%]	Defines/uses some correct input variable(s) with some incorrect input variable(s) [25%]	Defines/uses some correct input variable(s) but not all [50%]	Defines/uses input variables correctly [100%]
draw	Does not make any diagram or drawing [0%]	Makes a diagram or drawing that is not useful for identifying important variables [10%]	Makes a diagram or drawing with the potential to identify important variables but without being identified [30%]	Makes a diagram or drawing where you can identify some important variables but not all of those that such representation allows [70%]	Makes a drawing or diagram where the important variables can be identified [100%]
output	Does not identify any output variable [0%]	Defines/uses another variable(s) as output variables [10%]	Defines/uses some correct output variable(s) with some incorrect one(s) [25%]	Defines/uses some correct output variable(s) but not all [50%]	Correctly defines/ uses output variables [100%]
idsub	Does not identify any of the sub-problems [0%]	Identifies only some of the sub-problems [25%]	Identifies half of the sub-problems [50%]	Identifies almost all the sub-problems [75%]	Identifies all the sub-problems [100%]
solsub	Does not attempt to solve any of the sub-problems [0%]	Attempts but does not correctly solve any of the sub-problems [5%]	Solves some of the sub-problems correctly [50%]	Solves most of the sub-problems correctly [80%]	Solves all of the sub-problems correctly [100%]
intsub	Does not attempt to solve any of the sub-problems [0%]	Solves or attempts to solve only one of the sub-problems [5%]	Solves or attempts to solve some sub-problems in isolation or in incorrect order [50%]	Solves or attempts to solve most of the sub-problems and one of them is not in the correct order [80%]	Solves or attempts to solve all the sub-problems in the correct order [100%]

Variable	Indicators / Evidence				
norm	Does not attempt to resolve the problem [0%]	Solves or attempts to solve but does not declare at any time that the input variables must have the correct value and type [10%]	Solves or attempts to solve but does not declare for all input variables that they must have correct value and type [50%]	Solves or attempts to solve and explicitly declares for all input variables that they must have the correct value or type, but not both conditions [75%]	Solves or attempts to solve and explicitly declares for all input variables which must have correct value and type [100%]
extre	Does not attempt to solve the problem [0%]	Solves or attempts to solve without considering the extreme cases derived from the input variables [5%]	Solves or attempts to solve considering some of the extreme cases derived from the input variables [60%]	Solves or attempts to solve considering most of the possible extreme cases derived from the input variables [80%]	Solve or attempts to solve considering all possible extreme cases derived from the input variables [100%]
	or if there is only one extreme case:				
	Does not attempt to solve the problem [0%]	Solves or attempts to solve the problem but does not consider the extreme case [5%]	Identifies but does not attempt to solve the extreme case of the problem [25%]	Attempts but fails to solve the extreme case of the problem [50%]	Solves the problem considering the extreme case [100%]
flow	Does not attempt to solve the problem [0%]	Attempts to solve but does not respect the sequencing and does not use any selection or iteration structure being necessary [5%]	Attempts to solve respecting the sequencing but does not use any selection or iteration structure being necessary [40%]	Solves or attempts to solve using the necessary structures, but without respecting the indentation in case of using pseudocode, or without making the execution flow clear in case of using a flowchart [60%]	Solves or attempts to solve respecting the sequencing and using the necessary structures, with their appropriate indentation in case of using pseudocode, or with their clear execution flow in case of using a flowchart [100%]
syn	Does not attempt to solve the problem [0%]	Attempts to solve but does not use typical flowchart symbology, or does not use minimum pseudocode reserved words, such as If, Then, Else, End, Print, etc. [10%]	Attempts to solve using partially correct flowchart symbology, or partially clear pseudocode [40%]	Solves or attempts to solve using mostly correct flowchart symbology, or mostly clear pseudocode [80%]	Solves or attempts to solve using correct flowchart symbology, or completely clear pseudocode [100%]

Variable	Indicators / Evidence				
exe	Does not execute the algorithm [0%]	Executes the algorithm making many errors, mainly related to the execution flow [10%]	Executes the algorithm making few errors, mainly calculation of operations and less related to the execution flow [50%]	Executes the algorithm making a single error related to the calculation of operations and not related to the execution flow [80%]	Correctly executes the algorithm based on its representation [100%]
err	Does not execute the algorithm [0%]	Executes the algorithm by hand only for a combination of values of the input variables [10%]	Executes the algorithm by hand for some combinations of values of the input variables [40%]	Executes by hand the algorithm for various combinations of values of the input variables without identifying how to cover the main combinations [75%]	Executes the algorithm by hand for the main combinations of values of the input variables [100%]

Note. Translation of Table 3.4 from Navas-López (2021, pp. 61-65).

Navas-López (2021) also developed two specific rubrics for the two problems, the solution of which is an algorithm contained in the instrument used (p2 and p4). These two rubrics are versions derived from the one presented in Table 2 but adjusted to the particularities of both problems. Besides, both problems do not include the execution of the resulting algorithm due to time restrictions during the administration of the instrument, so they do not consider the component "Executing an algorithm" (from Table 1), that is, the *exe* and *err* variables. One of the problems did not require a graphic or diagram to be analyzed and solved, so it does not include the *draw* variable. So, observable variables assessed for problem p2 are: p2. input, p2.output, p2.idsub, p2.solsub, p2.intsub, p2.norm, p2.extre, p2.flow, p2.syn; and for problem p4 are: p4.input, p4.draw, p4.output, p4.idsub, p4.solsub, p4.intsub, p4.norm, p4.extre, p4.flow, p4.syn.

This research aims to construct validation for the specific rubrics applied by Navas-López (2021) as part of his operational definition for assessing AT in undergraduate students. Specifically, this construct validation will be carried out through CFA applied to several models proposed by the researcher based on the grouping of the measured variables for the two problems in the original measurement instrument.

Method

Problems p2 and p4, extracted from the instrument developed by Navas-López (2021), were administered to a sample of 88 undergraduate students enrolled in three academic programs offered by the School of Mathematics at the University of El Salvador. The participants, aged 17 to 33 (M: 20.88 years, SD: 2.509 years), included 41 women (46.59%), 46 men (52.27%), and 1 participant who did not report their gender. The overall grades of the subjects ranged from 6.70 to 9.43 on a scale of 0.00 to 10.00

(M: 7.64, SD: 0.57). The total student population at the time of data collection was 256, and a convenience sampling approach was employed during regular face-to-face class sessions across different academic years (first year, second year, and third year). The researcher verbally communicated the instructions for solving the problems, after which students individually solved them on paper, with the option to ask questions for clarification.

The translation of Navas-López's (2021) problem p2 is:

> A school serving students from seventh to ninth grade organizes a trip every two months, including visits to a museum and a theater performance. The school principal has established a rule that one responsible adult must accompany every 15 students for each trip. Additionally, it is mandated that each trip be organized by a different teacher, with rotating responsibilities. Consequently, each teacher may go several months (or even years) without organizing a trip, and when it is their turn again, they may not remember the steps to follow. Develop a simple algorithm that allows any organizer reading it to calculate the cost of the trip (to determine how much each student should contribute). (p. 70)

The translation of Navas-López's (2021) problem p4 is:

> As you know, most buses entering our country for use in public transportation have their original seats removed and replaced with others that have less space between them to increase capacity and reduce comfort.
>
> The company 'Tight Fit Inc.' specializes in providing this modification service to public transportation companies when they 'bring in a new bus' (which we already know is not only used but also discarded in other countries).
>
> Write an algorithm for the operational manager (the head of the workers) to perform the task of calculating how many seats should be installed and the distance between them. Assume that the original seats have already been removed, and the 'new' ones are in a nearby warehouse, already assembled and ready to be installed. Since the company is dedicated to this, it has an almost unlimited supply of 'new' seats. (p. 71)

To assess the students' procedures, the researcher employed the dedicated rubrics for these problems outlined by Navas-López (2021, pp. 77-98). Scores on a scale from 0 to 100 were assigned to the variables: p2.input, p2.output, p2.idsub, p2.solsub, p2.intsub, p2.norm, p2.extre, p2.flow, p2.syn, p4.input, p4.draw, p4.output, p4.idsub, p4.solsub, p4.intsub, p4.norm, p4.extre, p4.flow, p4.syn.

These 19 observable variables have been grouped in four different ways to construct the models for evaluation. The first group of models includes all 19 variables separately (p2&p4). The second group of models includes only the variables related to the first problem (p2). The third group of models includes only the variables related to the second problem (p4). Finally, the fourth group of models comprises intermediate variables obtained from the average of the corresponding variables in both problems (p2+p4), as follows:

- input:= (p2.input + p4.input)/2

- draw:= p4.draw

- output:= (p2.output + p4.output)/2

- idsub:= (p2.idsub + p4.idsub)/2

- solsub:= (p2.solsub + p4.solsub)/2

- intsub:= (p2.intsub + p4.intsub)/2

- norm:= (p2.norm + p4.norm)/2

- extre:= (p2.extre + p4.extre)/2

- flow:= (p2.flow + p4.flow)/2

- syn:= (p2.syn + p4.syn)/2

To assess the feasibility of conducting factor analysis on these four ways of grouping the observable variables, the researcher performs a data adequacy analysis. For assessing reliability, the Cronbach's alpha coefficient was utilized, and to evaluate construct validity, a CFA was applied to the models described in Table 3. All calculations were carried out using R language, version 4.3.2.

Table 3

Conformation of the evaluated factorial models

Model name	Factors (components)	Observed variables included
p2&p4_4c	Analysis	p2.input, p4.input, p4.draw, p2.output, p4.output
	Construction	p2.idsub, p4.idsub, p2.solsub, p4.solsub, p2.intsub, p4.intsub
	Cases	p2.norm, p4.norm, p2.extre, p4.extre
	Representation	p2.flow, p4.flow, p2.syn, p4.syn
p2&p4_5c	Analysis	p2.input, p4.input, p4.draw, p2.output, p4.output
	Decomposition	p2.idsub, p4.idsub
	Solution	p2.solsub, p4.solsub, p2.intsub, p4.intsub
	Cases	p2.norm, p4.norm, p2.extre, p4.extre
	Representation	p2.flow, p4.flow, p2.syn, p4.syn
p2_2c	F1*	p2.input, p2.output, p2.idsub, p2.solsub, p2.intsub
	F2**	p2.norm, p2.extre, p2.flow, p2.syn
p2_4c	Analysis	p2.input, p2.output
	Construction	p2.idsub, p2.solsub, p2.intsub
	Cases	p2.norm, p2.extre
	Representation	p2.flow, p2.syn
p2_5c	Analysis	p2.input, p2.output
	Decomposition	p2.idsub
	Solution	p2.solsub, p2.intsub
	Cases	p2.norm, p2.extre
	Representation	p2.flow, p2.syn
p4_4c	Analysis	p4.input, p4.draw, p4.output
	Construction	p4.idsub, p4.solsub, p4.intsub
	Cases	p4.norm, p4.extre
	Representation	p4.flow, p4.syn

Model name	Factors (components)	Observed variables included
p4_5c	Analysis	p4.input, p4.draw, p4.output
	Decomposition	p4.idsub
	Solution	p4.solsub, p4.intsub
	Cases	p4.norm, p4.extre
	Representation	p4.flow, p4.syn
p2+p4_2c	F1*	input, draw, output, idsub, solsub, intsub
	F2**	norm, extre, flow, syn
p2+p4_4c	Analysis	input, draw, output
	Construction	idsub, solsub, intsub
	Cases	norm, extre
	Representation	flow, syn
p2+p4_5c	Analysis	input, draw, output
	Decomposition	idsub
	Solution	solsub, intsub
	Cases	norm, extre
	Representation	flow, syn

Note. *General solution, **Particular cases and representation.

To evaluate the different models, the absolute fit indices chi-square (χ^2), relative chi-square (χ^2/df), RMSEA, SRMR, and the incremental fit indices TLI, CFI, NFI, and GFI were used. The evaluation was based on the respective cut-off values recommended by Jordan Muiños (2021) and Moss (2016), as presented in Table 4.

Table 4
Indices' cut-off values for Confirmatory Factor Analysis

	Indices							
	χ^2s p-value	χ^2/df	RMSEA	SRMR	TLI	CFI	NFI	GFI
Sample size < 100	≥0.05	<5	<0.05	<0.09	>0.9	>0.95	>0.95	>0.89
Sample size > 100	≥0.05	<5	<0.05	<0.08	>0.9	>0.95	>0.95	>0.93

Note. Own elaboration based on criteria from Jordan Muiños (2021) and Moss (2016).

Results

Of the 88 students, 82 attempted to solve problem p2 (6 did not attempt), 76 attempted to solve problem p4 (12 did not attempt), and 70 attempted to solve both problems. Everyone attempts to solve at least one problem. Cronbach's Alpha for problem p2 was 0.84, for problem p4 was 0.85, and for both problems (all data) was 0.88. Anderson-Darling test was applied to determine normality, and results showed no one observed variable are normal.

Kaiser-Meyer-Olkin measure was computed to assess the adequacy of the data for conducting a factor analysis. Additionally, Bartlett's test of sphericity was employed to determine whether there was sufficient correlation among the variables to proceed with a factor analysis. The scree test was also utilized to calculate the minimum number of recommended factors. This test defined the two-component models. The results presented in Table 5 reveal that there was a significant relationship among observable variables within four model groupings, supporting the feasibility of conducting a CFA.

Table 5
Results of data adequacy analysis for studied model groupings

Model grouping	KMO	Bartlett's χ^2	df	p-value	Suggested amount of factors by scree test
p2&p4	0.80	1012.4250	171	1.843e-119	5
Only p2	0.86	359.1103	36	6.814e-55	2
Only p4	0.78	346.5154	45	3.690e-48	3
p2+p4	0.87	465.5423	45	2.901e-71	2

For CFA, the WLSMV estimator (weighted least squares with robust standard errors and mean- and variance-adjusted test statistics) was used, following Brown's (2006) recommendations for ordinal, non-normal observable variables.

Corresponding indices were calculated for all models. Table 6 displays the calculated indices. All models achieve good values for incremental fit indices. However, models p2&p4_4c and p2&p4_5c fail in all absolute fit indices. Models p2_2c and p2+p4_2c do not successfully meet all absolute fit indices. For models p2_5c, p4_5c, and p2+p4_5c, all directly based on operationalization in Table 1, it was impossible to compute standard errors in CFA. Since standard errors represent how closely the model's parameter estimates approximate the true population parameters (Brown, 2006), all models with 5 components (factors) must be discarded.

Table 6

Results from the CFA conducted

Model	χ^2	df	p-value	χ^2/df	RMSEA	SRMR	TLI	CFI	NFI	GFI
p2&p4_4c	1749.13	146	<0.001*	11.980*	0.355*	0.366*	0.951	0.959	0.955	0.957
p2&p4_5c	1713.21	142	<0.001*	12.064*	0.357*	0.362*	0.951	0.959	0.956	0.958
p2_2c	44.910	26	0.012*	1.727	0.095*	0.093*	0.997	0.998	0.995	0.995
p2_4c	17.725	21	0.666	0.844	0.000	0.057	1.000	1.000	0.998	0.998
p2_5c**	17.301	17	0.434	1.018	0.015	0.057	1.000	1.000	0.998	0.998
p4_4c	31.441	29	0.345	1.084	0.034	0.071	1.000	1.000	0.996	0.997
p4_5c**	27.218	25	0.345	1.089	0.034	0.069	1.000	1.000	0.997	0.997
p2+p4_2c	47.184	34	0.066	1.388	0.067*	0.064	0.998	0.998	0.993	0.995
p2+p4_4c	17.630	29	0.951	0.608	0.000	0.041	1.000	1.000	0.998	0.998
p2+p4_5c**	16.014	25	0.914	0.641	0.000	0.040	1.000	1.000	0.998	0.998

Note. *Does not meet according to cut-off values in Table 4.

**Could not compute standard errors.

Only last three models with 4 components (factors) has very good psychometric properties. They are very similar to operationalization in Table 1, but in these models, decomposition-related observed variables (p2.idsub and p4.idsub) are placed together with algorithm-construction-related variables (p2.solsub, p2.intsub, p4.solsub and p4.intsub) inside the same factor. The factor loadings of these three models are shown in Table 7.

Table 7

*Factor loadings of observed variables in viable four-component models*

Component (factor)	Variable	p2_4c	p4_4c	Variable	p2+p4_4c
Problem analysis (requires abstraction): • Identify just-needed inputs and outputs. • Make a useful graphic or diagram.	p2.input	0.67		input	0.67
	p4.input		0.71		
	p4.draw		0.27	draw	0.44
	p2.output	0.85		output	0.88
	p4.output		0.85		
Construction of a correct algorithm that solves the problem: • Identify the sub-problems that compose the problem (also requires abstraction). • Implement a solution to the identified sub-problems. • Integrate the different partial solutions to solve the complete problem.	p2.idsub	0.94		idsub	0.93
	p4.idsub		0.95		
	p2.solsub	0.93		solsub	0.94
	p4.solsub		0.94		
	p2.intsub	0.98		intsub	0.94
	p4.intsub		0.91		

Component (factor)	Variable	p2_4c	p4_4c	Variable	p2+p4_4c
Identification of algorithm's input cases:	p2.norm	0.88		norm	0.89
	p4.norm		0.98		
• Identify normal cases	p2.extre	0.61		extre	0.69
• Identify special or extreme cases	p4.extre		0.98		
Formal representation:	p2.flow	0.85		flow	0.86
	p4.flow		0.85		
• Represent the execution flow.	p2.syn	0.98		syn	0.89
• Use appropriate symbology and syntactic rules.	p4.syn		1.00		

Figure 2 displays the structural equation modelling (SEM) diagram for the p2+p4_4c model, depicting factor loadings, residuals, and covariances between factors. Notably, only one-factor loading is relatively weak and pertains to the variable "draw." Conversely, all other factor loadings exhibit reasonably high values.

Figure 2

SEM diagram for validated model p2+p4_4c with four components for AT
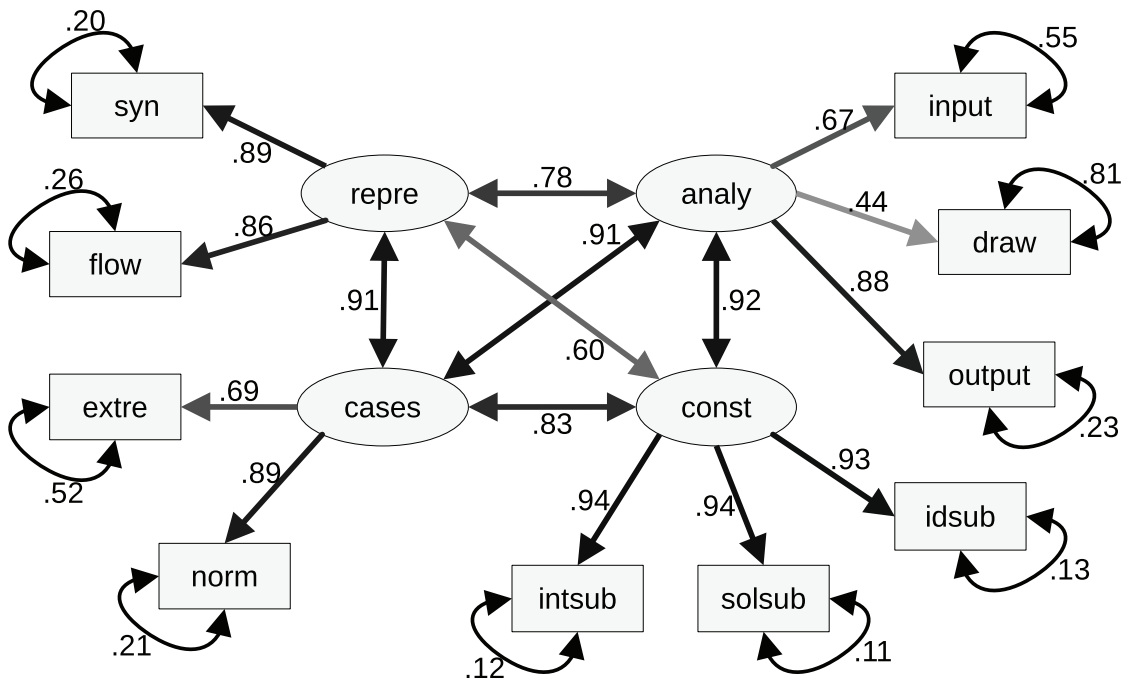
Discussion and conclusion

Grading consistency can be challenging, but rubrics serve as a standard scoring tool to reduce inconsistencies and assess students' work more efficiently and transparently (Chowdhury, 2019). Rubrics are crucial for evaluating complex cognitive skills like AT. Although analytic rubrics can slow down the scoring process because they require examining multiple skills individually (Mertler, 2001), their detailed analysis is valuable for understanding and developing AT (Lehmann, 2023).

Constructing reliable open-ended problems to assess AT is challenging due to the relation between problem complexity and students' prior experience. Thus, several considerations are essential: using a standard complexity metric for algorithms (Kayam et al., 2016), employing a general rubric for assessing CT activities (Otero Avila et al., 2019), and adapting problem complexity to students' levels (Bubica & Boljat, 2021).

The lack of consensus on the differences between CT and AT hinders the development of a standard rubric for AT. For instance, Bubica and Boljat's (2021) and Ortega Ruipérez and Asensio Brouard's (2021) interpretations of CT align with Stephens and Kadijevich's (2020) interpretation of AT. These diverse interpretations complicate the validation of their operational definitions.

Despite these difficulties, the design, construction, and construct validation of an assessment rubric for AT represent a valuable effort. Evaluating CT (Poulakis & Politis, 2021) and AT (Stephens & Kadijevich, 2020) remains an urgent concern for educational researchers. Therefore, any endeavor to advance towards a comprehensive operational definition of AT with both content and construct validity must be greatly appreciated.

In this study, the CFA conducted on undergraduate students (aged 17-33) led to an operational definition for AT composed of four components: Problem analysis, algorithm construction, input cases identification, and algorithm representation (as shown in Table 7). This result is confirmed by the good psychometric properties of the three four-factor models, with the two problems considered separately, p2_4c and p4_4c, and with the averaged results from both problems, p2+p4_4c (see Table 6).

These results provide more detail than the two-factor model (problem representation and problem-solving) by Ortega Ruipérez and Asensio Brouard (2021) for adolescents and the unifactorial model by Lafuente Martínez et al. (2022) for adults. However, comparability with Bubica and Boljat (2021) and Sung (2022) is limited, as both focus on children, and their models' factor loadings and psychometric properties are unsatisfactory. This may be due to children's ongoing cognitive development, as Sung (2022) notes. Indeed, higher-order thinking skills, such as complex problem-solving, develop as children transition to adolescence (Greiff et al., 2015).

However, there are significant limitations to consider. The sample size of 88 subjects is relatively small, potentially limiting the generalizability of the results. Moreover, the use of convenience sampling introduces bias, making it unclear if the sample represents all undergraduate students. These limitations underscore the need for cautious interpretation and highlight the necessity for future studies with larger, more diverse samples to validate these findings.

Furthermore, Navas-López's (2021) operational definition for AT lacks complete construct validation, particularly regarding the "running" component (see Table 1). Future research should address this by employing larger samples and using

a comprehensive instrument (including a broader rubric) that evaluates a range of problems, akin to p2 and p4, for grading the initial 10 variables, alongside proposed algorithms for the *run* and *err* variables (see Table 2).

In conclusion, while this study advances toward a detailed operational definition of AT, the sample size and sampling method constraints must be acknowledged. Continued research is crucial to strengthen the reliability and applicability of these findings, thereby facilitating the development of robust assessment tools for AT.

Notes:

Final approval of the article:
Verónica Zorrilla de San Martín, PhD, Editor in Charge of the journal.

Authorship contribution:
Eduardo Adam Navas-López: conceptualization, data curation, formal analysis, research, methodology, software, validation, visualization, writing of the draft and review of the manuscript.

Availability of data:
The dataset supporting the findings of this study is not publicly available. The research data will be made available to reviewers upon request.

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